UGRC 120
Numeracy Skills

Session 1

REVIEWS OF BASIC ALGEBRAIC MATHEMATICS II

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Session Overview

OVERVIEW

- Session 1 builds on Session 0. Here, we seek to introduce students to the concept of approximations and standard forms, solving problems involving rates and linear equations involving one or two unknown variables.

Session Outline

The key topics to be covered in the session are as follows:

- Approximations, Ratios, Proportions, Percentages and Rates
- Solving Linear Equations (One or Two Unknown Variables)
- Formulating and Solving Linear Equations

Goals and Objectives

At the end of the session, the student will be able to:

- Understand and compute basic measures of rates such as ratios, fractions, proportions and percentages.
- Perform basic approximations of numbers e.g. decimals and significant figures.
- Solve linear equations in one or two unknown variables.
- Formulate and solve linear equations from real world problems.

Topic One

APPROXIMATIONS, RATIOS, PROPORTIONS, PERCENTAGES & RATES

Section 1

In this section we shall discuss the following:

- Rates and Approximations
- Solving Linear Equations (One or Two unknown variables)
- Formulating and Solving Linear Equations (Real world problems)

Rates

and

Approximations

Ratios and Fractions

- A ratio expresses a comparison between two quantities. It is a relation between like numbers or values.

The ratio of *a* to *b* is written

- $a : b$ or
- $\frac{a}{b}$ or
- $a \div b$

RATES AND APPROXIMATIONS

- It can then be seen that a ratio is an indicated division. It gives an index of comparison.
- The first term, a, in the ratio is called the *antecedent* and the second term, b, is called the *consequent*.
- The quantities that are being compared should be expressed in the same units.

RATES AND APPROXIMATIONS

The following rules may be noted:

- A ratio can be reduced to its lowest terms:

$$\frac{5}{10} = \frac{1}{2} \text{ and } \frac{a^2}{ab} = \frac{a}{b}$$

- A ratio does not change if both its terms are multiplied by the same number:

$$\frac{2}{3} = \frac{4}{6} \text{ and } \frac{x}{y} = \frac{ax}{ay}$$

RATES AND APPROXIMATIONS

- The total of any quantity can be divided into parts corresponding to a given ratio by taking the fractional part of each term in the ratio of the whole.

  – To divide 10 marbles between two children in the ratio 2 : 3, we take the fractional parts $\frac{2}{2+3} = \frac{2}{5}$ and $\frac{3}{2+3} = \frac{3}{5}$ and give$10 \times \frac{2}{5} = 4$ marbles to the second child.

Rates and Approximations

Proportions

- A proportion is an equation expressing the equality between two ratios.

The equality of the ratios $\frac{a}{b}$ and $\frac{c}{d}$ may be written

- $a{:}b \;::\; c{:}d$
- $\frac{a}{b}\ or\ \frac{c}{d}$
- We may say that a, b, c, d are in proportion.

RATES AND APPROXIMATIONS

- The end terms a and d are called the $extremes$ of the proportion.
  - The middle terms b and c are called the $means$ of the proportion.

The following rules may be noted:

- The product of the means is equal to the product of the extremes.
- If $a : b :: c : d$ then $ad = bc$
- $1 : 2 :: 4 : 8$ then $1 \times 8 = 2 \times 4$

RATES AND APPROXIMATIONS

- Either the means or extremes or both can be interchanged:

  - If $\frac{a}{b} = \frac{c}{d}$ then $\frac{b}{d} = \frac{d}{c}$ or $\frac{a}{c} = \frac{b}{d}$ or $\frac{d}{b} = \frac{c}{a}$

- Refer to **"session 1 examples and activities" NOW** and take a look at Example 1.1
- Refer to **"session 1 examples and activities" NOW** and complete Activity 1.1

RATES AND APPROXIMATIONS

Direct Proportion and Inverse Proportion

- A direct proportion indicates a relationship between two quantities such that an increase in one leads to a corresponding increase in the other and a decrease in one leads to a corresponding decrease in the other.
  - *e.g. consider the relationship between the age of a child and the corresponding age of the father.*
  - *As age of son increases so does age of father, therefore there is a direct relationship between the age of a son and the corresponding age of a father.*

Rates and Approximations

- An inverse proportion indicates a relationship between two quantities such that an increase in one leads to a corresponding decrease in the other and vice versa.
  - E.g. the larger the number of competitors, the lower the price of the commodity.
  - This means that there is an inverse relationship between number of competitors and the price of a commodity.
  - This is true of course in general, but up to a certain limit.

Rates and Approximations

- Refer to **“session 1 examples and activities” NOW** and take a look at Example 1.2
- Refer to **“session 1 examples and activities” NOW** and complete Activity 1.2

Rates and Approximations

Percentages

- A percentage is the numerator of a fraction whose denominator is 100.
  - In other words it is a fractional or decimal part of 100.

- Five percent may be expressed in any of the following ways: $5\%, 0.05, \frac{5}{100}\ or\ \frac{1}{20}$

RATES AND APPROXIMATIONS

- Addition, subtraction, multiplication and division are done in a similar fashion as in numbers or fractions.

- A percent figure can be written either as a fraction or a decimal 50 percent can be written as $0.50, \frac{1}{2}$ or $\frac{50}{100}$

- Similarly, a decimal or a fraction can be written as a percentage i.e. 0.25 means 25%

RATES AND APPROXIMATIONS

- Refer to **“session 1 examples and activities” NOW** and complete Activity 1.3

RATES AND APPROXIMATIONS

- A rate is a comparison of unlike quantities whose units are given, e.g. kilometres per hour, number of lectures per day, etc.
  - To every rate, there is an associated fraction, e.g. 50 kilometres in 2 hours can be written

$$\frac{50\ kilometers}{2\ hours} = \frac{25\ kilometers}{1\ hour}$$

$$= 25\ km\ per\ hour$$

RATES AND APPROXIMATIONS

- Refer to **"session 1 examples and activities" NOW** and complete Activity 1.4

RATES AND APPROXIMATIONS

Rounding of Numbers and Significant Figures

- It is frequently desirable to abbreviate, or round numbers to make them easier to understand and use.
  - When doing this, however, we must be cautious about the results and their implications.
  - For example, if we are out shopping and we see some item on sale for GHC 9.99 most of us would view this as GHC 10, mentally, we round the original number to the nearest whole number.

Rates and Approximations

- The principle of rounding numbers is based largely on how many significant figures we require.

- The term significant figures relates to the number of digits in the number that are precise and accurate.

RATES AND APPROXIMATIONS

- The term significant figures relates to the number of digits in the number that are precise and accurate.
  - A budget figure of GH¢126, 784 contains six significant digits (all six numbers are accurate).
- GH¢ 127, 000 contains only three significant figures (the last three zeros are not).
  - Having decided we want the number to be rounded to three significant figures, we then take the last four digits of the original number and round to the nearest whole number: thus 6 784 becomes 7 000.

Rates and Approximations

- The reason for taking the last four digits is that we started with six significant figures, we require only three, so that the last $(3 + 1)$ digits need to be rounded.

- The only slight note of caution comes when rounding the number 5.
  - For example, if we had had 6500, should we round this to 7000 or to 6000?

- Our preference is that you round fives upwards to the nearest whole number.

Rates and Approximations

- Always round or approximate values after completing the arithmetic and not before and to ensure that the results which have been rounded are acknowledged as such.

Rates and Approximations

- Refer to **“session 1 examples and activities” NOW** and complete Activity 1.5

- Refer to **“session 1 examples and activities” NOW** and take a look at Example 1.3

- Refer to **“session 1 examples and activities” NOW** and complete Activity 1.6

Topic Three

SOLVING LINEAR EQUATIONS IN ONE AND TWO UNKNOWN VARIABLES

SOLVING LINEAR EQUATIONS IN ONE & TWO UNKNOWN VARIABLES

Introduction

- Relations between social and economic quantities can be expressed in the form of equations.
  - A better understanding of social systems can be obtained by a study of these relations usually expressed as systems of equations.
- A study of one or two simultaneous equations will be the subject of interest.

SOLVING LINEAR EQUATIONS IN ONE AND TWO UNKNOWN VARIABLES

Equations and Identities

- An equation is a mathematical statement showing the equality of two statements.
  - E.g. An equation in an unknown variable x. When a value of the variable is substituted in the equation making the equality true, then we have a solution or a root of the equation.

$x = 1$ is a root of the equation

- The roots of an equation can be found by applying suitable algebraic operations.

SOLVING LINEAR EQUATIONS IN ONE AND TWO UNKNOWN VARIABLES

- If an equation is true for all values of a variable, then it is called an *identity*.
  - We say the identity is true for all values of x.

- If an equality is not true for any value whatsoever of the variable, it is a *null equation*.

- If two equations have the same solution set, then they are said to *equivalent equations*.

SOLVING LINEAR EQUATIONS IN ONE AND TWO UNKNOWN VARIABLES

Degree of an Equation and its Solution

- The highest power of the unknown variable gives the degree of the equation.
  - A first degree or linear equation is a polynomial in one variable occurring only to the first power.
  - It is of the form $ax + b = 0$, where a and b are real constants.

- The solution is obtained by writing $ax = -b$ and getting

$$x = -\frac{b}{a}$$

Solving Linear Equations in One and Two Unknown Variables

- A second degree or a quadratic equation is of the general form $ax^2 + bx + c = 0$
  - Where *a* , *b*, and *c* are real valued constants. We obtain a solution of this equation by factorizing the LHS.

- It can be shown that the solution of the general quadratic equation is given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

  - This gives two roots or solutions of the quadratic equation.

Solving Linear Equations in One and Two Unknown Variables

- Refer to **“session 1 examples and activities” NOW** and take a look at Example 1.4

- Refer to **“session 1 examples and activities” NOW** and complete Activity 1.7

SOLVING LINEAR EQUATIONS IN ONE AND TWO UNKNOWN VARIABLES

Systems of Simultaneous Equations

- A linear equation in two unknown variables *x* , *y* is of the general form

$$ax + by = c$$

  - Where a , b , c are real valued constants.

- A solution of such an equation is an ordered pair such that $ax_1 + by_1 = c$
  - Which means that (x_1, y_1) satisfies the given equation.

Solving Linear Equations in One and Two Unknown Variables

- An equation with two unknown variables may have no solution or any number of solutions.

  – If there are two unknown variables, two equations are necessary to arrive at the solution.

Solving Linear Equations in One and Two Unknown Variables

Solving Simultaneous Linear Equations

- Solutions can be found for simultaneous equations only if the equations are consistent and independent.

- Simultaneous equations **should not be:**
  - Inconsistent as in $x + y = 3$,$x + y = 8$
  - Dependent as in $x + y = 3, 2x + 2y = 6$
- The second equation is derived from the first and does not provide an additional information about x , y .

Solving Linear Equations in One and Two Unknown Variables

Methods of Solving Simultaneous Equations

- Elimination by Addition and Subtraction
- Refer to **"session 1 examples and activities" NOW** and take a look at Example 1.5
- Elimination by Substitution

Refer to **"session 1 examples and activities" NOW** and take a look at Example 1.5

Solving Linear Equations in One and Two Unknown Variables

Graphical Method

- If there are two simultaneous equations in two unknown variables;
  - we can draw their graphs on the same rectangular coordinate system.
  - The point of intersection of the two graphs gives the solution to the simultaneous equations.

Solving Linear Equations in One and Two Unknown Variables

- Refer to **"session 1 examples and activities" NOW** and complete Activity 1.7

Topic Four

FORMULATING AND SOLVING LINEAR EQUATIONS

FORMULATING AND SOLVING LINEAR EQUATIONS

Formulating Linear Equations

- The first stage is to translate the problem into a mathematical formulation.

- Refer to **"session 1 examples and activities" NOW** and take a look at Example 1.6, 1.7

Reading List

- Refer to students to relevant text/chapter or reading materials you will make available on Sakai