# ECON 214
# Elements of Statistics for Economists

## Session 4 – Probability

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# Session Overview

- We begin the discussion of inferential statistics.
- Probability - computing the chance that something will occur in the future - underlies statistical inference.
- That is, the drawing of conclusions from a sample of data. If samples are drawn at random, their characteristics (such as the sample mean) depend upon chance.
- Hence to understand how to interpret sample evidence, we need to understand chance, or probability.

# Session Overview

- At the end of the session, the student will
  - Be able to define probability and describe the classical, empirical and subjective approaches to probability
  - Understand the terms: experiment, event, outcome, and sample space
  - Calculate probabilities applying the rules of addition and multiplication
  - Use a tree diagram to organize and compute probabilities
  - Determine the number of outcomes in an experiment using the multiplication, permutation and combination formulas

# Session Outline

The key topics to be covered in the session are as follows:

- Defining probability
- Calculating probability
- Addition rule of probability
- Multiplication rule of probability
- Determining number of outcomes

# Reading List

- Michael Barrow, “Statistics for Economics, Accounting and Business Studies”, 4th Edition, Pearson
- R.D. Mason , D.A. Lind, and W.G. Marchal, “Statistical Techniques in Business and Economics”, 10th Edition, McGraw-Hill

Topic One

# DEFINING PROBABILITY

# Defining probability

- If samples are drawn at random, their characteristics (such as the sample mean) depend upon chance.
- Hence to understand how to interpret sample evidence, we need to understand chance, or probability.
- Simply put, **probability is the chance or likelihood that something or an event will happen**.

# Defining probability

- The probability of an event *A* may be defined in two ways:
  - The **objective** (or **frequentist**) **view**: the proportion of trials in which the event occurs, calculated as the number of trials approaches infinity
  - The **subjective view**: someone's degree of belief about the likelihood of an event occurring.

# Defining probability

- The objective view of probability in turn may be divided into (1) classical probability and (2) empirical probability.

# Defining probability

- **Classical view** assumes that all outcomes of an experiment are equally likely and mutually exclusive, and that the probability of an event occurring is the ratio of the number of outcomes to the sample space.
  - By mutually exclusive, we mean the occurrence of any one outcome precludes the occurrence of any other in the same trial.
  - In the classical approach, the probability of an event is known "a priori."
  - **Example:** for a fair die, the probability of any number appearing in a single toss is $^1/_6$.

# Defining probability

- In the **empirical** (or relative frequency) approach, the probability is determined on the basis of the proportion of times that a particular event occurs in a set of trials.
- This approach is called empirical because it is based on the collection and analysis of data.
- The probability value obtained in both classical and empirical approaches indicate the long run rate of occurrence of the event (that is, when the experiment is performed a large number of times).

# Some terminologies

- **Experiment:** an activity such as tossing a coin, which has a range of possible observations or outcomes.
- **Outcome:** a particular result of an experiment.
- **Trial:** a single performance of the experiment.
- **Sample space:** all possible outcomes of the experiment. For a single toss of a coin the sample space is {Heads, Tails}.
- **Event:** a collection of one or more outcomes of an experiment.

Topic Two

# CALCULATING PROBABILITY

# Calculating probability

- Let *A* be the event we want to calculate a probability for, and *S*, the sample space.
- The probability of the event A occurring is given by

$$\mathrm{P}(A) = \frac{n(A)}{n(S)}$$

- Where n(A) is the number of times the event ***A*** occurs and n(S) is the sample space.
- Example: On a single toss of a coin, P (Heads) = ½ ; P (Tails) = ½. Why?

# Rules of probabilities

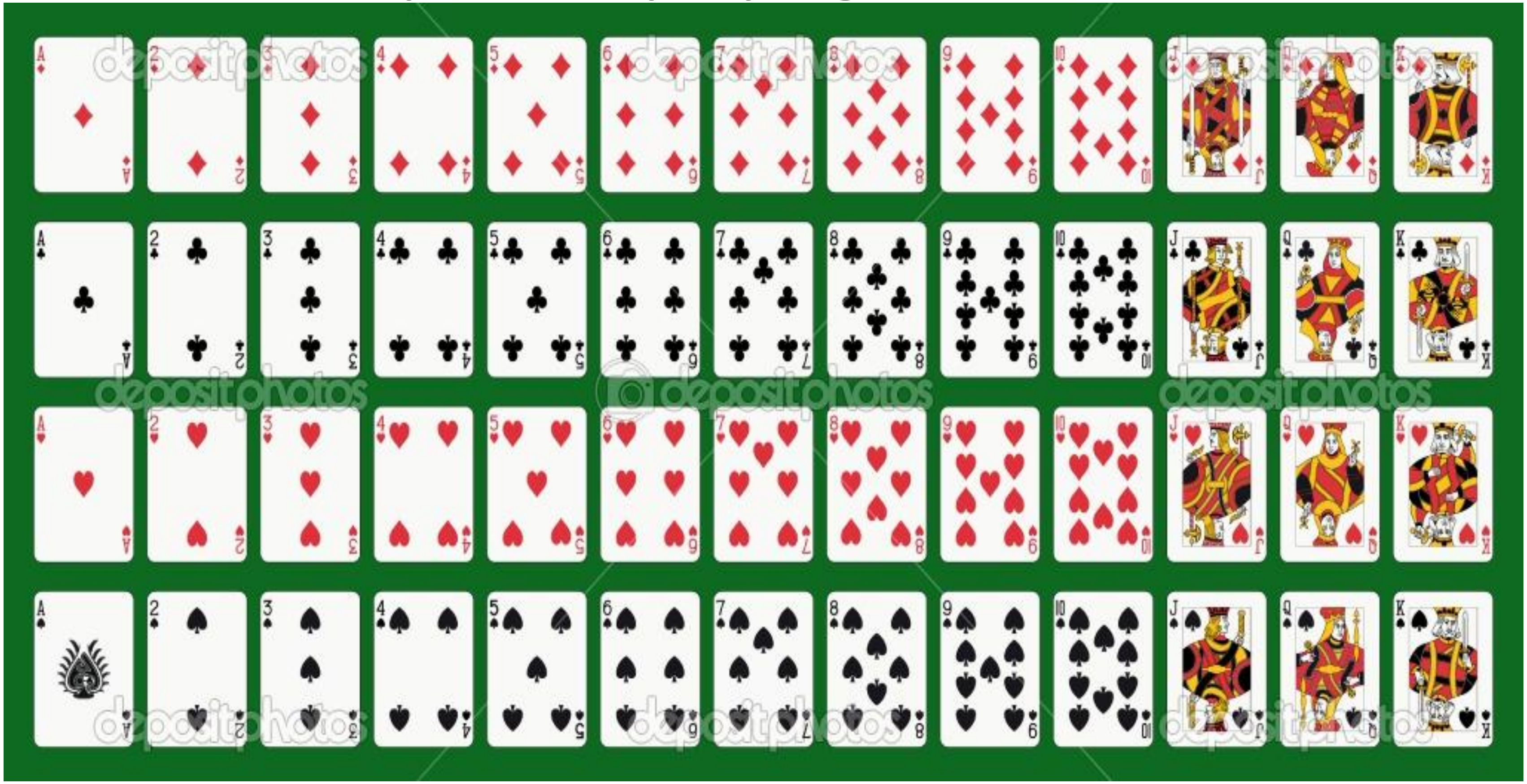
- The probability of an event lies between zero and 1, i.e. $0 \leq P(A) \leq 1$

- $\sum P(A_i) = 1$; i.e. summed over all outcomes

- P (not $A$) = 1 - P($A$)
- P (not $A$) is called the complement of $A$.

# Illustration

- Consider a pack of playing cards



# Illustration

- The probability of picking any one card from a pack (e.g. King of Spades) is 1/52.
- This is the same for each card since there are 52 cards in all.
- Summing over all cards: $1/52 + 1/52 + ... + 1/52 = 1$
- P (not King of Spades) = 51/52 = 1-P(King of Spades)

Topic Three

# ADDITION RULE OF PROBABILITY

# Compound events

- Often we want to calculate more complicated probabilities:
  - what is the probability of drawing any Spade?
  - what is the probability of throwing a ‘double six’ with two dice?
  - what is the probability of a randomly chosen student from this class obtaining exam grade in ECON 214 better than a B?
- These are **compound events** because they involve more than one outcome.
- A student making a grade better than a B must either make a B+ or an A.

# Mutually exclusive and non-exclusive events

- **Mutually exclusive**: two or more events that cannot occur together
  - Example: rolling a die and finding the probability of 4 or 5 showing up.
- **Non-exclusive**: two or more events that can occur together
  - Example: picking a king of hearts from a pack of cards.

# Addition rule for probabilities

- The addition rule is used when we wish to determine the probability of either one event or another (or both).
- The addition rule: the **'or'** rule
  - P(*A* or *B*) = P(*A*) + P(*B*)
    The probability of rolling a five or six on a single roll of a die is

| | | | | | |
|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 |

  - P(5 or 6) = P(5) + P(6) = 1/6 + 1/6 = 1/3
  - This is the (special) addition rule for mutually exclusive events.

# Addition rule for probabilities

- If *A* and *B* can simultaneously occur (i.e., they are non-exclusive), the previous formula gives the wrong answer...
  - P (King or Heart) = 4/52 + 13/52 = 17/52 ✖
  - This double counts the King of Hearts; 16 dots highlighted

| | A | K | Q | J | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Spades | • | • | • | • | • | • | • | • | • | • | • | • | • |
| Hearts | • | • | • | • | • | • | • | • | • | • | • | • | • |
| Diamonds | • | • | • | • | • | • | • | • | • | • | • | • | • |
| Clubs | • | • | • | • | • | • | • | • | • | • | • | • | • |

# Addition rule for probabilities

- We therefore subtract the King of Hearts:
  So P(King or Heart) = 4/52 + 13/52 - 1/52 = 16/52
- The formula is therefore
  – P(*A* or *B*) = P(*A*) + P(*B*) - P(*A* and *B*)
- This is the general rule for the addition of probabilities (whether mutually exclusive or non-exclusive events)
- When *A* and *B* cannot occur simultaneously (mutually exclusive), then P(*A* and *B*) = 0, and we obtain the special rule discussed earlier.

Topic Four

# MULTIPLICATION RULE OF PROBABILITY

# Dependent and Independent Events

- **Independent events**: Means the occurrence (or non-occurrence) of one event has no effect on the probability of occurrence of the other
  - Example: tossing a coin twice and obtaining two heads.
- **Dependent events**: Means the occurrence (or non-occurrence) of one event does affect the probability of occurrence of the other
  - Example: drawing two aces from a pack of cards, one after the other, without replacement.

# The multiplication rule

- Multiplication rule is used when we want probability that all of several events will occur.
- That is, when you want to calculate P(*A* and *B*):
- $P(A \text{ and } B) = P(A) \times P(B)$
- This is the (special) multiplication rule for **independent** events
- Example: probability of obtaining a double-six when rolling two dice is: $P(6 \text{ and } 6) = P(6) \times P(6) = {}^1/_6 \times {}^1/_6 = {}^1/_{36}$.

# Multiplication rule

- There is a slight complication when events are dependent.
- For instance, P(drawing two Aces from a pack of cards, without replacement)...
- If the first card drawn is an Ace ($P = {}^{4}/_{52}$), that leaves 51 cards, of which 3 are Aces.
- The probability of drawing the second Ace is ${}^{3}/_{51}$, different from the probability of drawing the first Ace.
- They are not **independent events**. The probability changes.
- Thus P(two Aces) = ${}^{4}/_{52} \times {}^{3}/_{51} = {}^{1}/_{221}$

# Conditional probability

- $^{3}/_{51}$ is the probability of drawing an Ace **given** that an Ace was drawn as the first card.
- This is the **conditional probability** and is written

  P (Second Ace | Ace on first draw)

  – That is, the probability of a second Ace given that an Ace was drawn first.
- In general, it is written as **P(B|A)**
- That is, the probability of event B occurring, given A has occurred.

# Conditional probability

- Consider P (A2| not-A1)...
- A 'not-Ace' is drawn first, leaving 51 cards of which 4 are Aces
- Here, we assumed that the first card drawn is **not** an Ace.
- Hence P (A2| not-A1) = $^{4}/_{51}$
- So P (A2| not-A1) $\neq$ P (A2|A1)
- They are **not** independent events.

# Conditional probability

- The general rule for multiplication is

  $$P\ (A \text{ and } B) = P\ (A) \times P\ (B|A)$$

- For independent events

  $$P\ (B|A) = P\ (B|\text{ not-}A) = P\ (B)$$

- And so

  $$P\ (A \text{ and } B) = P\ (A) \times P\ (B)$$

Topic Five

# DETERMINING NUMBER OF OUTCOMES

# Combining the rules

- Consider that we wish to find the probability of one head in two tosses of a coin.
- We can solve this by combining the addition and multiplication rules
- P (1 Head in two tosses)...
- ...= P **(** [H and T] or [T and H] **)**

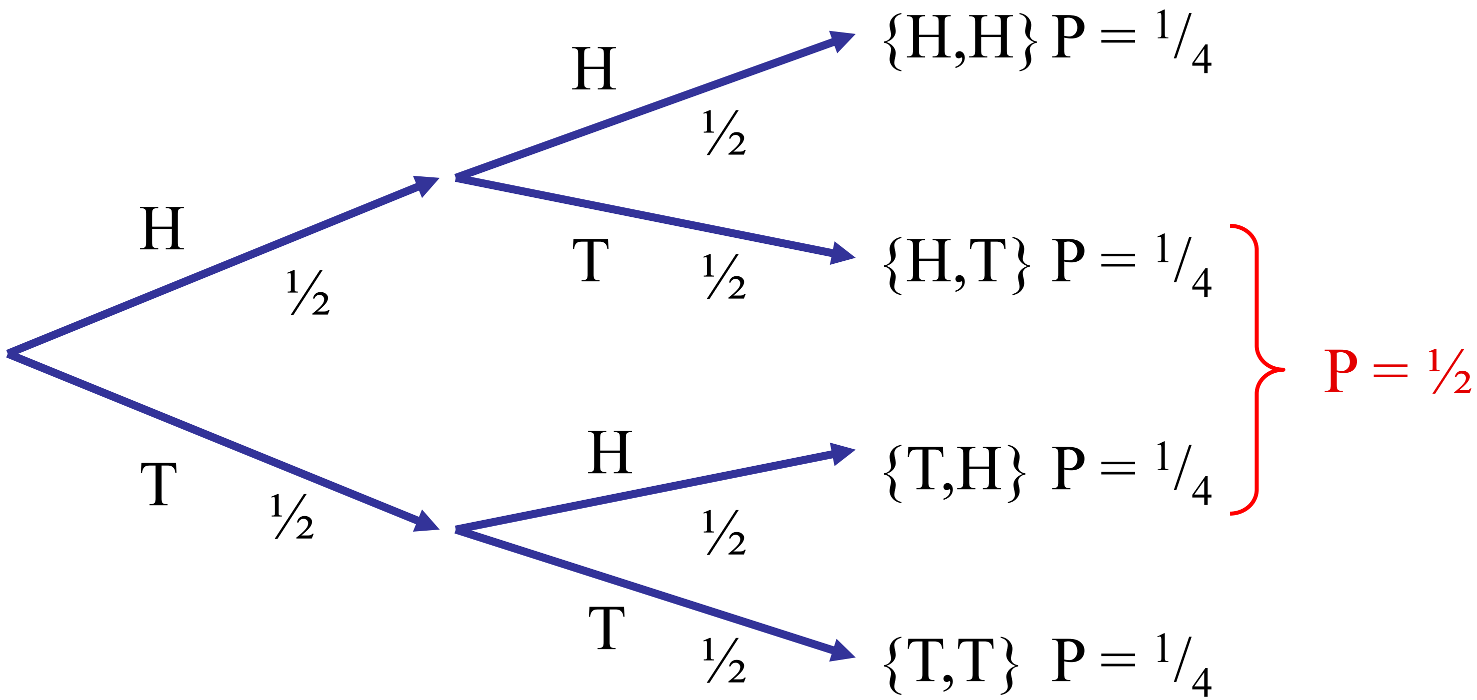
  = P **(** [H and T] **)** + P **(** [T and H] **)**

  = $[1/2 \times 1/2] + [1/2 \times 1/2]$

  = $1/4 + 1/4 = 1/2$

# Tree diagram

- We can also solve the problem using a tree diagram



# Gets complicated with larger number of outcomes

- What about calculating
  - P (3 Heads in 5 tosses)?
  - P (30 Heads in 50 tosses)?
- How many routes through the tree diagram?
  - Drawing takes too much time, so we need a formula...
- In other words, if the number of possible outcomes in an experiment is large, it is difficult or cumbersome to list the total number of outcomes in either the event set or sample space.
- There are techniques for determining the number of outcomes.

# Multiplication formula

- The multiplication formula is used to find the total number of outcomes for two or more groups of objects.
- If there are $n_1$ objects of one kind and $n_2$ of another, then there are $n_1 \times n_2$ ways of selecting both.
- That is, total number of outcomes = $n_1 \times n_2$
- In general if there are *k* groups of objects, and there are $n_1$ items in the first group, $n_2$ items in the second, …, and $n_k$ items in the $k^{th}$ group, the number of ways we can select one item each from the *k* groups is: $n_1 \times n_2 \times \ldots\ldots\ldots \times n_k$
- If $n_1 = n_2 = \ldots\ldots\ldots = n_k$, then $n_1 \times n_2 \times \ldots\ldots\ldots \times n_k = n^k$

# Permutation formula

- The permutation formula is applied to find the number of outcomes when there is only one group.
- We are interested in how many different subsets that can be obtained from a given set of objects.
- Example - the number of ways of having 3 girls in a family of 5 children.
- If ***r*** items are selected from a set on ***n*** objects (where $r \leq n$), any particular sequence of these *r* items is called a permutation.

# Permutation formula

- The formula is

$$nPr = \frac{n!}{(n-r)!}$$

- Where $n! = n \times (n-1) \times (n-2) \times \ldots \times 1$

- n = total number of objects
- r = number of objects selected at a time
- n! is called "n factorial" and it is the product of all the integers up to and including n.

# Permutation

- So the number of different ways of having 3 girls in a family of 5 children is

$$5P3 = \frac{5\times4\times3\times2\times1}{2\times1} = 60$$

- In permutation the <u>order</u> of arrangement of the objects is <u>important</u>!
- For example, the arrangement (Kofi, Ama) is a different permutation from (Ama, Kofi) even though it is the same two individuals.

# Combination formula

- Just like permutation, the **combinatorial formula** gives the number of ways in which a particular event may occur, but **<u>without</u>** regard to order.
- The formula for combination is

$$nCr = \frac{n!}{r!(n-r)!}$$

- Again $n! = n \times (n-1) \times (n-2) \times \ldots \times 1$

# Combination formula

- So 3 girls in a family of 5 children is

$$5C3 = \frac{5\times 4\times 3\times 2\times 1}{(3\times 2\times 1)\times(2\times 1)} = 10$$

# Combination

- We can write the probability of 1 Head in 2 tosses as the probability of a head and a tail (in that order) times the number of possible orderings (# of times that event occurs).
- P (1 Head) = $\frac{1}{2} \times \frac{1}{2} \times 2C1 = \frac{1}{4} \times 2 = \frac{1}{2}$
- We can formalise this in the Binomial distribution ….. soon!

# References


- Michael Barrow, “Statistics for Economics, Accounting and Business Studies”, 4th Edition, Pearson
- R.D. Mason , D.A. Lind, and W.G. Marchal, “Statistical Techniques in Business and Economics”, 10th Edition, McGraw-Hill