PSYC 331 STATISTICS FOR PSYCHOLOGIST Session 3 –INFERENCES ABOUT THE MEAN OF A SINGLE AND TWO POPULATIONS

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Session Overview

- This session introduces leaners to two important statistical tests that are used by social scientists when they want to draw inferences from a single and double populations.
- The goal of this session is to equip students with the ability to:
- Differentiate between a parametric statistical test and a nonparametric statistical test;
- analyze data that has been collected from a single population when the population standard deviation is known as well as when the population standard deviation is not known;
- analyze data that that has been collected from two independent populations as well as analyze data collected from two dependent populations; and answer any question in inferential
 Dr. P. N. DOKU, Slide 2

Session Outline

- The key topics to be covered in the session are as follows:
- Inferences about the mean of a single population when the population standard deviation is known – the Z test.
- Inferences about the mean of a single population when the population standard deviation is not known – the *t* test based on a single population.
 - •Steps to follow in answering an examination question in inferential statistics
 - Inferences about the difference between the means of two
 - unrelated populations the independent *t* test.
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Reading List

- Opoku, J. Y. (2007). Tutorials in Inferential Social Statistics. (2nd Ed.). Accra: Ghana Universities Press. *Pages 45 – 84*
- MiĐhael Baddoll, StatistiĐs fod EĐoŶoŵiĐs, AĐĐouŶtiŶg aŶd BusiŶess Studies, 4th Edition, Pearson
- R.D. MasoŶ, D.A. LiŶd, aŶd W.G. MadDhal, StatistiDal TeDhŶidues iŶ BusiŶess aŶd EĐoŶoŵiĐs, ббth Edition, McGraw-Hill



The difference between a parametric and a nonparametric test: Brief review

•A parametric test involves the estimation of at least one population parameter and makes assumptions about the shape of the distribution. It is used when measurement on the dependent variable is at least on an interval scale.

•A nonparametric test does not make any assumptions about the shape of the distribution in the population. It is used when the scale of measurement is on an ordinal or nominal scale. nonparametric test has alsogot more prover than and The difference between a parametric and a nonparametric test: Brief review

•Since the aim of any researcher is to reject the null hypothesis Ho and support the research (working) hypothesis H_A or H_1 , it is always advisable to use a parametric test to analyze data, provided that the assumptions underlying the use of a selected parametric test are satisfied.

•This is because in any research, the researcher states Ho with the aim of probability of jectering hockwheeption gistiplicand a Parametric test does exactly that — it has a

TOPIC ONE: INFERENCES ABOUT THE MEAN OF A SINGLE POPULATION WHEN THE POPULATION STANDARD DEVIATION IS KNOWN – THE Z TEST

- Recall that a Z-score can measure the location of a given value on a normal distribution, which can be expressed as a probability.
- A Z-test uses the normal distribution to obtain a *test statistic* based on some data that can be compared with a *sampling distribution of chance*, which is an abstract construction drawn from the data.
- The *z*-test is the procedure for computing a *z*-score for a

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WHEN TO USE THE Z TEST

- The Z test is used to make an inference from a single sample to a single population when both the mean and standard deviation of scores in the population are known.
- Also the data should be measured on at least an interval scale.



Z-TEST ASSUMPTIONS

• An important assumption underlying the use of the *Z* test is that the sampling distribution of data in the population is continuous and normal in form.

We have randomly selected one sample

The dependent variable is at least approximately normally distributed in the population and involves an interval or ratio scale

We know the mean of the population of raw scores under another condition of the independent variable

We know the true standard deviation of the population (σ_X) described by the null hypothesis



Z-DISTRIBUTION





A Two-Tailed Test

- Create the sampling distribution of means from the underlying raw score population that H₀ says our sample represents
- Choose the criterion, symbolized by α (alpha)
- Locate the region of rejection which, for a twotailed test, involves defining an area in both tails
- Determine the critical value by using the chosen α to find the $z_{\rm crit}$ value resulting in the appropriate region of rejection



A Two-Tailed Hypothesis

- In a two-tailed test, the null hypothesis states the population mean equals a given value. For example, H_0 : μ = 100.
- In a two-tailed test, the alternative hypothesis states the population mean does not equal the same given value as in the null hypothesis. For example, H_1 : $\mu \neq 100$.



A Sampling Distribution for H_0 Showing the Region of Rejection for $\alpha = 0.05$ in a Two-tailed

Figure 7.4

Sampling Distribution of IQ Means for a Two-Tailed Test

A region of rejection is in each tail of the distribution, marked by the critical values of \pm 1.96.



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Computation of Z observed

• The *z*-score is computed using the same formula in the previous slide



Is the standard error of the mean based on the population



Zobs vs Zcrit

- In a two-tailed test, reject H_0 (and obviously support H_1) if the *z*-score you computed is
 - Less than or equal to the negative of the critical zvalue

OR

- Greater than or equal to the positive of the critical
- *z*-value
- Otherwise, fail to reject H_0



Rejecting vs Retaining H_o

REJECTING H_o

- •When the z_{obs} falls beyond the critical value, the statistic lies in the region of rejection, so we reject H_0 and accept H_1 .
- •When we reject H_0 and accept H_1 we say the results are **significant**. Significant indicates the results are unlikely to occur if the predicted relationship does not exist in the population.

• RETAINING H_o

- When the z_{obs} does not fall beyond the critical value, the statistic does not lie within the region of rejection, so we do not reject H₀.
- When we fail to reject H₀ we say the results are nonsignificant. Nonsignificant indicates the results are likely to occur if the predicted relationship does not exist in the population.



Z-test- some points to Note

- If Z is large, the probability that H₀ is true is small
- If Z (test statistic) is larger than critical value, then H₀ is rejected
- If Z (test statistic) is smaller than critical value, then H₀ is not rejected (failure to reject null)

Note when results are nonsignificant:

- When we fail to reject H_0 , we do not prove H_0 is true
- Nonsignificant results provide no convincing evidence the independent variable does not work



One-Tailed Hypotheses

In a one-tailed test, if it is hypothesized the independent variable causes an *increase* in scores, then the null hypothesis states the population mean is less than or equal to a given value and the alternative hypothesis states the population mean is greater than the same value. For example:

$$H_0: \mu \le 50(any)$$

 $H_1: \mu > 50(any)$

In a one-tailed test, if it is hypothesized the independent variable causes a *decrease* in scores, then the null hypothesis states the population mean is greater than or equal to a given value and the alternative hypothesis states the population mean is less than the same value. For example:





One-Tailed Hypotheses: sampling distribution for region of rejection of

re 7.7

ng Distribution of IQ Means for a One-Tailed Test of Whether Scores Increase of rejection is entirely in the upper tail.

Figure 7.8

Sampling Distribution of IQ Means for a One-Tailed Test of Whether Scores Dec The region of rejection is entirely in the lower tail.



Illustration Example

Use the following data set and conduct a twotailed *z*-test to determine if μ = 11 and the population standard deviation is known to be 4.1

14	14	13	15	11	15
13	10	12	13	14	13
14	15	17	14	14	15

Illustration Example

1. $H_0: \mu = 11; H_1: \mu \neq 11$ 1. Choose $\alpha = 0.05$ 2. Reject H_0 if $z_{obt} > +1.96$ or if $z_{obt} < -1.96$.

$$\sigma_{\bar{x}} = \frac{\sigma_{\bar{x}}}{\sqrt{N}} = \frac{4.1}{\sqrt{1.8}} = 0.966$$

$$z_{obs} = \frac{\overline{X} - \mu}{\sigma_{\overline{X}}} = \frac{13.67 - 11}{0.966} = 2.764$$

Since z_{obs} lies within the rejection region, we reject H_0 and accept H_1 . Therefore, we conclude μ does not equal 11.

One or Single Sample t Test

 What is the purpose of the One/Single sample t test?

the one complet test??

• How is it different from the Z test?

• What are the assumptions underlying

What is the Purpose of a Single-Sample t-Test?

• Compare a sample mean to a hypothesized population mean.

- Test the Null Hypothesis that the sample came
- from a population with a particular mean.

• Use for a one-sample design.



How is it Different from a z-Test?

The z test requires you to know the σ, but you usually doŶ't kŶo∥ it.

• If you doŶ't kŶo $\| \sigma$, your best estimate of it is s_x .

• When you use s_x instead of σ , you are doing a t-test.



Comparing z and t

• Z test(one sample)



t test(one sample)







How is it Different?

The t distribution is symmetrical but flatter than a normal distribution.

•The exact shape of a *t* distribution depends on degrees of freedom

Unlike the Z test, the t value is influenced by sample size so that **for every sample size there is a different t distribution**. The t tables are therefore read with reference to **degrees of freedom** (df), a concept which is related to sample size



How is it Different?

normal distribution



t distribution

What Are Degrees of Freedom?

- DEFINITION: The number of degrees of freedom fod aŶy statistiĐ is the Ŷuŵd'ed of ĐoŵpoŶeŶts iŶ its ĐalĐulatioŶ that ade fdee to |ady;MĐ Call, 6∈ΘG, p. 222)
- It is an information related to sample size
- Changes depending on the design and statistic

- For a one-group design, df = N-1
- The last sĐođe is Ŷot fdee to |ady



What are the

Independent observations.

Interval or ratio level data.

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t-DISTRIBUTION





Computing the One-Sample t-Test

- 1. Compute the estimated sample standard deviation $\left(S^{2}\right)_{X}$ using the formula $S^{2}_{X} = \frac{\sum X^{2} - \frac{(\sum X)^{2}}{N}}{N-1}$
- 2. Compute the **estimated standard error of the mean** ($S_{\overline{X}}$) using the formula $s_{\overline{X}} = \frac{S_X}{\sqrt{M}}$
- 3. Calculate the t_{obs} statistic using the formula

$$t_{\rm obs} = \frac{X - \mu}{s_{\overline{X}}}$$



t-test

After calculating the value of t using the formula , the observed t value is referred to the t tables with the appropriate degrees of freedom. Like the Z test, the t_{obs} is compared to the critical value(s) of t at the selected decision rule (level of significance) and a decision is taken as to whether to retain or reject H.



Using the *t*-Table (Critical Values)

Obtain the appropriate value of t_{crit} from the *t*-tables using

- The correct table depending on whether you are conducting a one-tailed or a two-tailed test,
- The appropriate column for the chosen α , and
- The row associated with your degrees of freedom (*df*)



Example

Use the following data set and conduct a two-tailed *t*-test to determine if $\mu = 12$

14	14	13	15	11	15
13	10	12	13	14	13
14	15	17	14	14	15



Example

- $H_0: \mu = 12; H_a: \mu \neq 12$
- Choose α = 0.05
- Reject H_0 if $t_{obt} > +2.110$ or if $t_{obt} < -2.110$

$$t_{\text{obt}} = \frac{X - \mu}{\frac{S_X}{\sqrt{N}}} = \frac{13.67 - 12}{\frac{1.61}{\sqrt{18}}} = \frac{1.67}{0.380} = 4.40$$

• Since 4.4 > 2.110, reject H_0 and conclude μ does not equal 12



TOPIC 3: GENERAL STEPS TO FOLLOW IN ANSWERING A QUESTION IN INFERENTIAL STATISTICS (REVIEW)

- 1. Choice of statistical test
- 2. Statement of hypotheses
- 3. Decision rules
- 4. Computation
- 5. Decision
- 6. Interpretation



TOPIC 4: Inferences about the mean of two populations: overview

- Two different samples of participants are compared on a single outcome score OR Participants' sDodes ade weasuded uŶded t||o conditions of the independent variable
- Condition 1 produces sample mean $\begin{bmatrix} & X_1 \\ & \end{bmatrix}$ representing μ_1

Figure 9.1

Relationship in the Population in a Two-Sample Experiment As the conditions change, the population tends to change in a consistent fashion.



Overview cont;

 The parametric statistical procedure for determining whether the results of a twosample experiment are significant is the twosample t-test

- The two versions of the two-sample *t*-test are
 - The independent-samples t-test
 - The correlated-samples t-test



INFERENCES ABOUT THE DIFFERENCE BETWEEN THE MEANS OF TWO UNRELATED POPULATIONS – THE INDEPENDENT t TEST

- •We are collecting two sample means and finding out how big the difference is between them.
- •The mean of this sampling distribution is the H_o difference between population means, which is zero.
- •Two completely different (independent) groups of subjects that you want to compare to determine if they are significantly different from one another: a between-groups design.
- •E.g. Experimental and control conditions.
- •Compare the means of the two different conditions/groups.



When to use Independent t test



Figure 4.1: A diagramatic representation of 2 samples drawn from male(A) and female(B) populations. (It is to be noted that $\mathbf{I}_{1}^{\mu}, \overline{\mathbf{X}}_{1}$; and $\mathbf{I}_{2}^{\mu}, \overline{\mathbf{X}}_{2}$ are arbitrarily defined to represent the male and the female populations and samples respectively. One may also use \mathbf{I}_{1}^{μ} and $\overline{\mathbf{X}}_{1}$ and \mathbf{I}_{2}^{μ} and $\overline{\mathbf{X}}_{2}$ to refer to the respective female and male populations and samples)

Independent Samples t test Assumptions

1.Two samples and a between-groups design (two independent groups).

2. The sampling distribution is normally distributed.

3.The dependent variable is on an interval or ratio scale

4. The variances of the two groups are the same or are homogeneous.

The Standard Error of the Difference

We may conceptualize the standard error of the difference (Se_{diff}) between two population means **as a standard deviation of the difference between two sample means** drawn from the two populations.

$$se_{diff.}(s_{\bar{x}_1-\bar{x}_2}) = \sqrt{\left[\sum X_1^2 - \frac{(\sum X_1)^2}{n_1} + \sum X_2^2 - \frac{(\sum X_2)^2}{n_2}\right] \left[\frac{n_1 + n_2}{n_1 n_2 (n_1 + n_2 - 2)}\right]}$$



The independent t-test formula



 It should be noted that the sign (-/+) of the t in the formula depends on the numerator only since the denominator always gives a positive value

Degrees of freedom associated with the t and critical values

Critical values for the independent samples t-test (t_{crit}) are determined based on

- •degrees of freedom $df = (n_1 1) + (n_2 1)$,
- •the selected α , and
- •whether a one-tailed or two-tailed test is used

•In a two-tailed *t*-test of independent samples, reject H_0 if t_{obs} is greater than or equal $\ge t_{crit}$ or if t_{obs} is less than or equal to the - t_{crit}

•Otherwise, fail to reject H_0

Degrees of freedom associated with the t and critical values

df = (n1 + n2 - 2)

The calculated value of t from formula is referred to the t tables with the appropriate degrees of freedom [=(n1 + n2 - 2)] under a specified decision rule and a decision is taken as to whether H_0 should be retained or rejected



Statement of hypothesis: Two tailed(non-directional)

- For a two-tailed test, the statistical hypotheses are
- =0 $H_0: \mu_1 - \mu_2$

 $H_1: \mu_1 - \mu_2 \neq 0$

- H_o implies both samples represent the same population of scores
- H₁ implies the means from our conditions each represent a different population of scores



Statement of hypothesis: Two tailed(non-directional)

The statistical hypotheses for a one-tailed test of independent samples are

$$\begin{aligned} H_0 : \mu_1 & -\mu_2 & \leq 0 & H_0 : \mu_1 & -\mu_2 & \geq 0 \\ H_1 : \mu_1 & -\mu_2 & > 0 & OR & H_1 : \mu_1 & -\mu_2 & < 0 \end{aligned}$$

If μ_1 is expected to be larger than μ_2

If μ_2 is expected to be larger than μ_1



Points to note about a One tailed test

- Conduct one-tailed tests only when you can confidently predict the direction the dependent scores will fall.
- 2. Decide which X and corresponding µ is expected to be

larger

- 3. Arbitrarily decide which condition to subtract from the other
- 4. Decide whether the difference will be positive or negative
- 5. Create H_1 and H_0 to match this prediction
- 6. Locate the region of rejection



Using the following data set, conduct an independent-samples *t*-test. Use $\alpha = 0.05$ and a two-tailed test.

	Sample	1		Sample	2
14	14	13	15	11	15
13	10	12	13	14	13
14	15	17	14	14	15



Worked example

• Use the six steps in the previous sessions

$$t_{\text{obs}} = \frac{(\overline{X}_{1} - \overline{X}_{2})}{S_{X-X_{1-2}}}$$
$$= \frac{(13.556)}{-13.778)}$$
$$= -0.285$$
$$0.780$$



Worked example

• t_{crit} for df = (9 + 9 - 2) = 16 with $\alpha = .05$ and a two-tailed test is 2.120 (**refer to t-tables**).

• Reject H_0 if t_{obs} is greater than +2.120 or if t_{obs} is less than -2.120.

• Because t_{obs} of -0.285 is not beyond the $-t_{crit}$ of -2.120, it does not lie within the rejection region. We retain H_0 . INFERENCES ABOUT THE DIFFERENCE BETWEEN THE MEANS OF TWO RELATED POPULATIONS – THE CORRELATED t TEST

> Also called: Dependent Samples or Related Samples t-test or paired t test

- Compares two conditions with paired scores:
- -Within subjects design
- -Matched groups design
- -Identical twins



When to use the correlated t Test

- You have two measures on the same subjects
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 . The same subjects can serve as their own control.
- You have two separate samples but the subjects in each are individually matched so that there are similar subjects in each group (but not the same subjects in each group).
- Identical twin studies (naturally matched)



Paired Samples t-Test Assumptions

^{1.} The sampling distribution is normally distributed.

2. The dependent variable is on an interval or ratio scale



Paired sample t test-example

 Table 9.1: An example of d'efode aŶd afted paidiŶg using the same subjects in eaĐh paided saŵple

11	11	M	u	
10	11			

Subject	Score Before Treatment	Score After Treatment
1	50	55
2	52	58
3	44	48
4	42	41
5	49	56

Table 9.2: An example of a
 paided desigŶ iŶ ||hiĐh the actual subjects in each sample ade diffedeŶt d'ut ade ŵatĐhed for characteristics which they have in common (in this example, genetics)

14		4	۸		
л		۵	ч		
	11			14	

Twin Pair	Twin 1 = Control Group	Twin 2 = Experimental Group
A	10	8
В	12	10
С	21	19
D	18	15



The standard error of the mean for correlated samples





Correlated t Test formula

$$t_{obs} = \frac{\overline{D} - \mu_D}{S_D^-} \text{ Simplified to } t = \frac{\sum D}{\sqrt{\frac{n \sum D^2 - (\sum D)^2}{n-1}}}$$

The computed value of t (t_{obs}) is referred to the t tables under a specified decision rule with df=(n - 1) and a decision is taken as to whether H_o should be retained or rejected in either a one-tailed or two-tailed test of significance.

Statement of hypotheses

Two tailed

 $H_0 : \mu_D = 0$ $H_1 : \mu_D \neq 0$

One tailed H₀: $\mu_D \leq 0$ H₁: $\mu_D > 0$

• OR H₀ : μ_D \geq 0 H₁ : μ_D D < 0



Degrees of freedom

- Since we are dealing with a single population (the population of difference scores), the degrees of freedom (df) associated with this t test = n 1, where n is the sample size, which is equal to the number of pairs of scores. df=n-1
- In a two-tailed test of related samples, reject H_0 if t_{obs} is greater than $+t_{crit}$ or if t_{obt} is less than $-t_{crit}$
- Otherwise, retain H₀



WORKED EXAMPLE

Question

- To determine the influence of early childhood social environment on emotional adjustment at adolescence, 16 sets of identical twins orphaned during infancy were obtained. For each twin pair, one member was placed in a foster home while the other member grew up with relations of their natural parents. At the age of 18, all twins were administered a self—report inventory that measured degree of emotional adjustment (DEA). [Scores could range between 10 and 80 with higher scores reflecting better adjustment].
- From the data in the table below, did the type of early childhood social environment have any influence on the emotional adjustment of the twins at adolescence?

Degrees of freedom

Twin Pair	DEA score of twin raised	DEA score of twin raised	
	in foster home	by relations of natural	
		parents.	
1	50	55	
2	50	60	
3	60	45	
4	65	55	
5	70	60	
6	40	40	
7	58	52	
8	62	64	
9	68	60	
10	45	50	
11	56	44	
12	60	57	
13	70	63	
14	35	38	
15	66	54	
16	59	Slide 61 51	

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Solution: Step 1: Choice of Statistical Test

- •We are given two sets of scores that are correlated with each other.
- The two populations involved may be reduced to a single population of difference scores (*D scores*).
 We assume that the distribution of difference scores is normal within the population.
- •We further assume that the scores on emotional adjustment have at least been measured on an interval scale.

•Therefore, the most appropriate statistical test to use to answer the question is the **matched or correlated** *t* **test**.

Step 2: Statement of Hypotheses

- We are required to determine whether or not the type of early childhood upbringing has an influence on emotional adjustment at adolescence. The hypotheses should be 2-tailed since we are not told the direction of the possible influence of early childhood social environment on emotional adjustment at adolescence.
- Let μ_D represent the population mean of difference scores. Then, the null hypothesis (H_o) and the alternative hypothesis (H₁) may be stated as:
- $\rm H_{o}\,$: The type of early childhood social environment has no significant influence on

emotional adjustment of the orphaned twins at adolescence ($\mu_D = 0$).

• H_1 : The type of early childhood social environment has a significant influence on

emotional adjustment of the orphaned twins at adolescence ($\mu_D \neq 0$).



Step 3: Decision Rules

Given: 0.05 level of significance, a two-tailed correlated t test, df = n - 1 = 16 - 1 = 15, the critical values of t from the t tables = ± 2.131 .

Therefore;

- Retain H_o if -2.131 < t_{obs} < 2.131, and
- Reject H_o if t_{obs} 4 -2.131 or if t_{obs} \pm 2.131



Step 4: Computations

• With the usual notations, the computational formula for the correlated t test is given by:

$$t = \sum D / \sqrt{\frac{n \sum D^2 - (\sum D)^2}{n - 1}}$$

- NB//: Compute all the necessary summations and input in the formula (show workings in examination) $t = \frac{17,568 4,356}{15}$
 - $t_{obs} \approx \uparrow. \uparrow \uparrow \varkappa$; corrected to f_{slide} are splaces?



Step 5: Decision

- Referring (t_{obs} = 2.224) to the decision rules in (step 3), we note that 2.224 > 2.131.
- Therefore, H_o is rejected at the 0.05 level of significance.



Step 6: Interpretation

- At the 0.05 level of significance, a significant difference exists between the emotional adjustment scores obtained by twins raised by foster parents and those raised by family relations.
- Therefore, the type of early childhood upbringing had a significant influence on the emotional adjustment of the twins at adolescence.



CRITICAL POINT TO NOTE

 Use these same steps in answering ALL your statistics question from z test, t test, independent t test, the one way ANOVE etc. including the non parametric testS.



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