PSYC 441
Cognitive Psychology II

Session 10 – Thinking and Problem Solving

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Session Overview

- The topic is about discussing different kinds of thinking and problem solving which is concerned about the kind of mental work we do every day. What are the general problems we face? How do we manage to accomplish such complex tasks? What processes do we use? These are important issues that we will discuss in this section.

Session Objectives

- At the end of the session, the student will be able to

- Define the cognitive process of thinking
- Describe problem solving as a cognitive process
- Discuss the characteristics of problems and their classifications
- List problems and their methods of solutions

Session Outline

The key topics to be covered in the session are as follows:

- Topic One: Scope of Thinking and Problem Solving
- Topic Two: Classification of Problems
- Topic Three: Problems and Methods of Solution
- Topic Four: Specific Problem Solving Strategies

Reading List

- Ashcraft, M. H. (2013). Cognition (6th edn.), London: Pearson Education Int.
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Topic One

SCOPE OF THINKING AND PROBLEM SOLVING

Overview

Overview

Human beings are generally problems solvers and we solve variety of problems all the time.

- Examples of problems
  - *"I want to earn a first class at the University, what study plan should I use?".*
  - *"I want Mary to notice me at the party, how should I arrange for it?"*
  - *"I am trying to research into a social problem, how do I do it based on the stated premises?"*

☞ Some problems are trivial, but solving them have implications for our survival and social recognition.

- **Thinking and Problem Solving**

- Thinking is a ***polysemous*** concept therefore a precise definition is a very difficult task since it also requires thinking.

Some definitions of thinking:-

- **Bruner (1957)** describes thinking as *"going beyond the information given".*
- **Barlett (1958)** defines thinking as *"a complex and high evel skill that fills up gaps in the evidence".*

- **Baron (1994)** defines it as "*what we do when we are in doubt about how to act, what to believe, or what to desire".*

- **Newell and Simon (1972)** have also described thinking ***"as a process of searching through a problem space".***

- Clearly, **thinking** is used to refer to more than one specific activity, suggesting that there may be different types of thinking.

- One way is to consider thinking as either ***focused*** *or* ***unfocussed*** *thinking.*

  ♦ **Focused thinking –** begins with a clear starting point and has a specific goal.

  ♦ **Unfocused thinking –** has the character of daydreaming, or unintentionally calling to mind a number of different and loosely related ideas.

  The topic of **thinking** is traditionally divided into a number of more specific topics including **problem solving**, **reasoning**, and **decision making and judgement.**

Problem Solving

We will focus only on **Problem Solving** for now

PROBLEM SOLVING

A problem exists when a living organism has a goal but does not know how this goal is to be reached.

Karl Dunker, 1945

Problem Solving

PROBLEM SOLVING

- ♦ Researchers have found it important to compare problem-solving to a search process – *as if you were navigating through a maze, looking for a path to enable you reach a goal.*

- Some of the paths in the maze will lead you to realize the goal of getting out, while others will lead to dead ends or to realize wrong ends.

– Some of the paths in the maze are direct, others are long and overstretched while others may be blocked.

Problem Solving

- Problem solving as **a search process** was central to the thinking of Newell and Simon (1972).

- Problem-solving starts with an ***initial*** or ***start state*, ** which includes the knowledge and resources you have at the outset, which helps you to work toward a goal state.

- In addition, the problem-solver has a set of operators *(tools or actions)* that can change his current state. The operators help the problem-solver to move from the initial state to the goal or end state).

Problem Solving

- In the process, there is likely to be a set of **Path Constraints** which represent blocks to reaching the goal.
- Now if you consider the ***initial state***, the ***operators*** and the ***constraints***, there could also be a number of intermediate constraints that may lead to the creation of **sub-goals,** en route to the main goal.
- The processes described constitute the **Problem Space.**
- Problem-solving then is ***a search through the problem space***. A large problem space (from start to finish) can make solution to a problem difficult while a small problem space enhances problem solution.

Topic Two

CLASSIFICATION OF PROBLEMS

Classification of Problems

CLASSIFICATIONS OF PROBLEMS

– Researchers have found it convenient to classify problems faced by problem solvers. Especially important is the distinction between **Well-defined** and an **Ill-defined** problems (Reitman, 1965). There are others as well.

Well-defined Problems

– A well-defined problem is one in which the initial and goal state as well as the operators and actions needed to move from one state to another can be specified.

– ***Such problems have:***

1 . Clearly defined start state
2 . Clearly defined goals

Classification of Problems

- In a well-defined problem, every proposed solution can be evaluated against the criteria implied by the goal. If the proposed solution matches the criteria implied by the goal, the problem is solved.

- A correct answer exists for a well-defined problem. In word puzzles, an **anagram** (*word or phrase rearranged to form another word or phrase)* is a good example of a well-defined problem.

Example:

- If you are asked in an anagram problem solving to rearrange **TAENCEN** into a word, the initial state is the configuration of the letters. Try this **BNEMCCEUEANR.**

- By applying operators and rearranging the letters, the goal state is derived.

Classification of Problems

Examples of well-defined problems:

- What is the value of the unknown in the equation:
  48 = *y* / 3.2

- Rearrange the following to form a word –

- **LSSTNEUIAMYOUL, OTNUSOOYLERMCPENA**

Three common types of well-defined problems (Greeno,1978).

1. Problems of Inducing Structure
2. Problems of Transformation
3. Problems of Arrangement

Classification of Problems

1. Problems of Inducing Structure

- Such problems require a person to discover a pattern that will relate elements of the problem to each other. (i.e., look for elements of the problem and how they relate to each other). An example is that of verbal analogies such as:

a. Chimney is to a house as -------- is to ship.
b. University of Ghana is to Accra as .... is to Kumasi

- Solving this problem you must understand that a chimney is a structure designed to allow smoke and carbon to escape safely from a fireplace in a house.

- With this knowledge, a problem solver can properly produce *smokestack* as an analogous structure for a ship and KNUST as analogous for Kumasi.

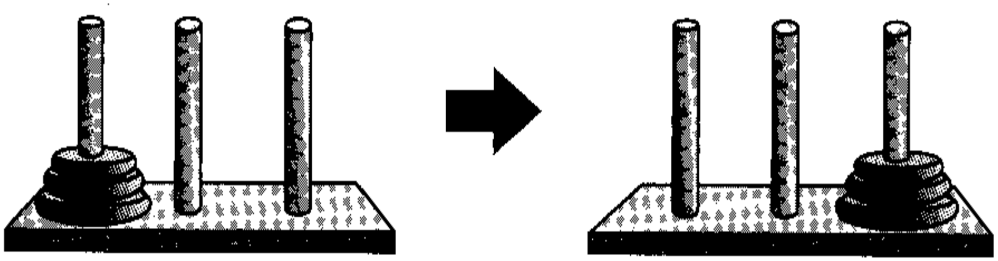
  (Lack of Vocabulary may limit your ability to respond)

Classification of Problems

2. Problems of Transformation

- Here, a person must **manipulate** objects or symbols in accordance with certain rules in order to obtain a solution.

An example is the well-known **Tower of Hanoi problem.**



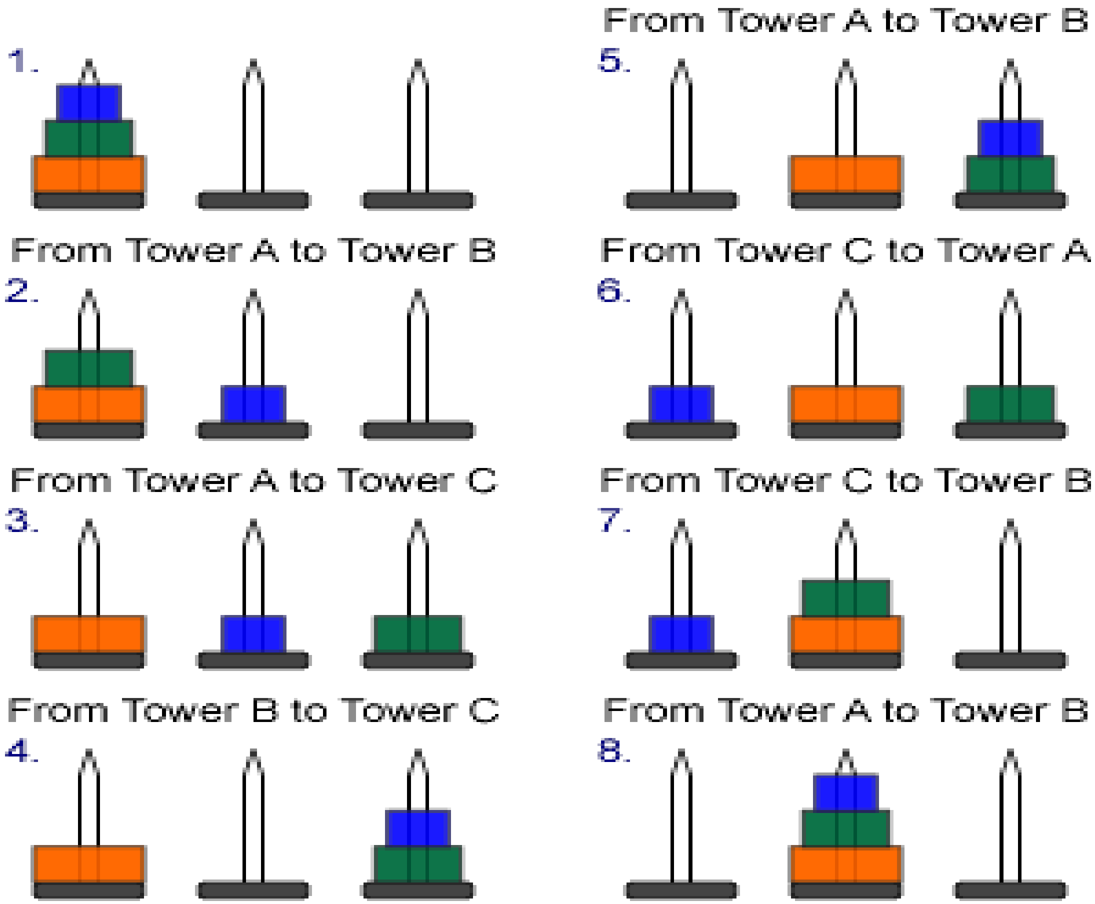
Classification of Problems

- The three-disk version of the Tower of Hanoi problem

Instructions:

- Transfer the three disks from the first peg to the third peg, moving only one disk at a time and never placing a bigger disk on top of a smaller one. Try to determine the number of moves until the goal state is achieved.

Solution


1.
From Tower A to Tower B
2.
From Tower A to Tower C
3.
From Tower B to Tower C
4.
From Tower A to Tower B
5.
From Tower C to Tower A
6.
From Tower C to Tower B
7.
From Tower A to Tower B
8.

Classification of Problems

3. Problems of Arrangement

- This problem involves **rearrangement** of elements of a task in order to solve a problem. All of the elements of such tasks are given and the problem solver must rearrange the elements in some way that solves the problem.

The elements are rearranged.

- A typical example of problems of arrangement is demonstrated using anagrams, and the principal cognitive skill needed to solve such a problem is that of **constructive search**.

Classification of Problems

- For example, how many words can be formed from the two anagrams **GANRE, TARIL .**
- Based on research by Mayer (1983) it took an average response time of 8 sec to solve for RANGE, but took an average of about 114sec to respond to ANGER.

ILL-Defined Problem

- Ill-defined or ill-structured problems have components of the problem space (either initial or goal states or operators or some combination) not specified or are incomplete.
- There may also be more than one "correct" answer.
- For example buying a car, renting an apartment are examples of ill-defined problems.

Classification of Problems

- These are ill-defined because the goal state (the car purchased or the house rented), is not always known at the beginning of the problem.

- Although the problem solver may have an ideal model to buy, he or she may settle for a compromise based on a number of factors.

- We usually select what is “best” and what is best may be defined in terms of location, cost, and amenities available. We call this ***satisficing***.

- What we may consider best at any time may not be good enough compared with well-defined problems which have clear starting states and well defined goal states.

Classification of Problems

- While a lot of problems studied in the laboratory are well-defined, many of the problems we face in life are ill-defined.

- In ill-defined problems, a person is likely to use some type of **heuristics** in solving the problem but such heuristics are less likely to provide guarantees for solving the problems.

Summary – ill-defined.

- We encounter ill-defined problems on daily basis and our inability to solve these problems in a rational and systematic way can be attributed to several factors including:

    a. The solution is not always clear

    b. We are willing to accept the best solution based on convenience – known as *satisficing*

    c. Heuristics are used and are often driven by our beliefs and schemata which may not always be completely accurate.

Classification of Problems

Routine or Non-routine Problems

- Problems are also classified as **Routine** or **Non-routine.**
- **A Routine Problem** involves the use or application of operators in a predictable, systematic manner known to the problem solver.
- Multiplying 2986 by 3434 is a routine problem. You only need to follow the rules of multiplication and addition toward the solution of the problem.
- **A Non-routine Problem** requires the problem solver to apply operators in a novel fashion or use a procedure that is not well known to the problem solver.
- Most of the research problems in the social sciences are based on non-routine problems. Solving such problems is not systematic and predictable.

Classification of Problems

– It requires a lot of thinking and innovation to solve such problems. Insights gained during problem solving situations involve largely non-routine applications.

Adversary and Nonadversary

– Another useful distinction for problems is that of **Adversary** and **Nonadversary** problems.

- **Adversary** problems involve competition between two or more players. Chess and most competitive games are examples. Problem solvers are competitors, who adopt variety of strategies to win, defeat or outmanoeuvre their opponents.
- **Nonadversary** Problems do not require the problem solver to face a competitor and can exert a lot of control over the problem space than those in adversary types of problems e.g., fixing your car, writing novels etc.

Topic Three

PROBLEMS AND METHODS OF SOLUTION

Algorithism

Problems and Methods of Solution

- People attempt to solve problems by using some kind of strategies, which represent a systematic attack to a problem.
- We distinguish between two kinds of general strategies **algorithms** and **heuristics.**
- **Algorithm** is a set of rules or procedures which ensures the solution. It follows a specified sequence of steps that guarantees a solution if one exists.
- E.g., an algorithm for solving a six-letter anagram (e.g., **socure**) would be to vary the order of the six letters systematically and look up each permutation in a dictionary until a match is found (e.g., course, source).

Algorithms

- You will generate all **720** permutations until a solution is achieved.
- **Algorithms** are used in computer problem solving. But is the computer really thinking? It uses algorithm and blindly following a sequence of steps at a lightening speed.
- How many of you will use algorithm to check the 720 permutations?
- No one because we rely on strategies known as **heuristics** when solving a problem.

Heuristic

- A **heuristic** strategy by contrast, is a rule of thumb or approximation which may or may not ensure the solution. People rely on heuristics because they often provide a solution <u>faster</u> than in algorithm. E.g., searching for a friends number in the phone book.

☞ ***If algorithmic strategies ensure solution, why do we use a heuristic rule of thumb when an algorithm is guaranteed to work?***

Heuristics

The reasons are that:

- Algorithmic strategies are quite cumbersome and time-consuming.
- They are sometimes very expensive.
- Many problems we face as human beings do not have an algorithms *(e.g., guessing an answer during exams, piloting a plane when you have not been trained).*

Topic Four

SPECIFIC PROBLEM SOLVING APPROACHES

Specific Problem Solving Approaches
s

- Algorithms and heuristics are strategies that can be considered at the general level. However, there are specific problem solving strategies that are identified, these may fall into any of the two broad categories discussed earlier.

☞ These are some of the specific approaches to problem solving::

- Generate-and-test technique
- Means-ends analysis
- Working backward
- Backtracking
- Reasoning by analogy
- Hill-climbing heuristic

Generate-and-Test Technique

Generate-and-test Technique

- It consists of generating possible solutions to problems and testing them. This technique loses its effectiveness very rapidly when there are many possibilities and when there is no particular guidance over the generation process.

  E.g., if you forget the combination numbers used to open your padlock, searching can be so frustrating that you may give up *(the problem space may be too large).*

- However, where there aren't many possibilities the generate-and-test technique may prove useful. This is similar to the **trial-and-error** process in which solutions are tried and discarded until one that works is discovered.

Means-ends Analysis

Means-ends Analysis

- Requires people to determine the ends they wish to achieve and the means by which they will reach these ends.

- There is the possibility of setting sub-goals *(if the duration is long)* which gradually lead toward the final desired goal.

  ♦ For example, If your goal is to obtain a First Class honours, that constitutes the end, then, you will have to determine the means to reach the end, e.g., sub goals, taking lectures seriously, discipline, study plan and so on.

Working Backward

Working Backward

- The problem solver is required to analyse the goal to determine the last step needed to achieve it, then the next-to-last step and so on.
- For instance, in the problem of getting to your friend’s house, the very last step is to walk from outside her front door into the house.
- The problem of getting to her front door can be solved by taking a taxi to her house. You can get a taxi at the Gate etc.
- Certain mathematical problems cannot be solved unless you resort to working backward.

Working Backward Assignment

Consider the following problem by Wickelgren (1974).

- Three people play a game in which one person loses and two people win each game. The one who loses must double the amount of money that each of the other two players has at that time. The three players agree to play three games. At the end of the three games, each player has lost one game, and each player has eight dollars.

What was the original stake of each player? **Take it as assignment.**

- The problem can be frustratingly difficult unless you work from the goal state, which in this problem is the only known state.

- Working backward often involves establishing subgoals so its functions are similar to means-ends analysis but in a reversed order in this case.

Backtracking

Backtracking

- This method requires that you make provisional assumptions before you start.
- In several instances, the assumptions turn out to be wrong and must be "unmade" or corrected.
- In such situations the assumptions need to be reviewed to enable you resolve the problems.
- Sometimes new assumptions have to be made or abandoned altogether. *Cul-de-sac.*

Reasoning by Analogy

Reasoning by Analogy

- This is the use of prior knowledge and experience to solve new problems or simply, we draw on analogies.
- Problem solving in this situation is based on perceived similarities between the prior experience and the new problem.
- E.g., when you write an exam in social psychology, you may use many of the strategies that were helpful when you wrote a paper in environmental psychology. In that situation, we are using analogical reasoning.

☞ Now read the following passages and see if they are related in any way.

Duncker Tumor Problem

- **Passage 1 (Tumor Problem)**

  ☞ Suppose you are a doctor faced with a patient who has a malignant tumor in his stomach. It is impossible to operate the patient, but unless the tumor is destroyed the patient will die. There is a kind of ray that can be used to destroy the tumor. If the rays reach the tumor all at once at a sufficiently high intensity, the tumor will be destroyed. Unfortunately, at this intensity the healthy tissue that the rays pass through on the way to the tumor will also be destroyed. At lower intensities the rays are harmless to healthy tissue, but they will not affect the tumor either.

- What type of procedure might be used to destroy the tumor with the rays, and at the same time avoid destroying the healthy tissue? (Duncker,1945).

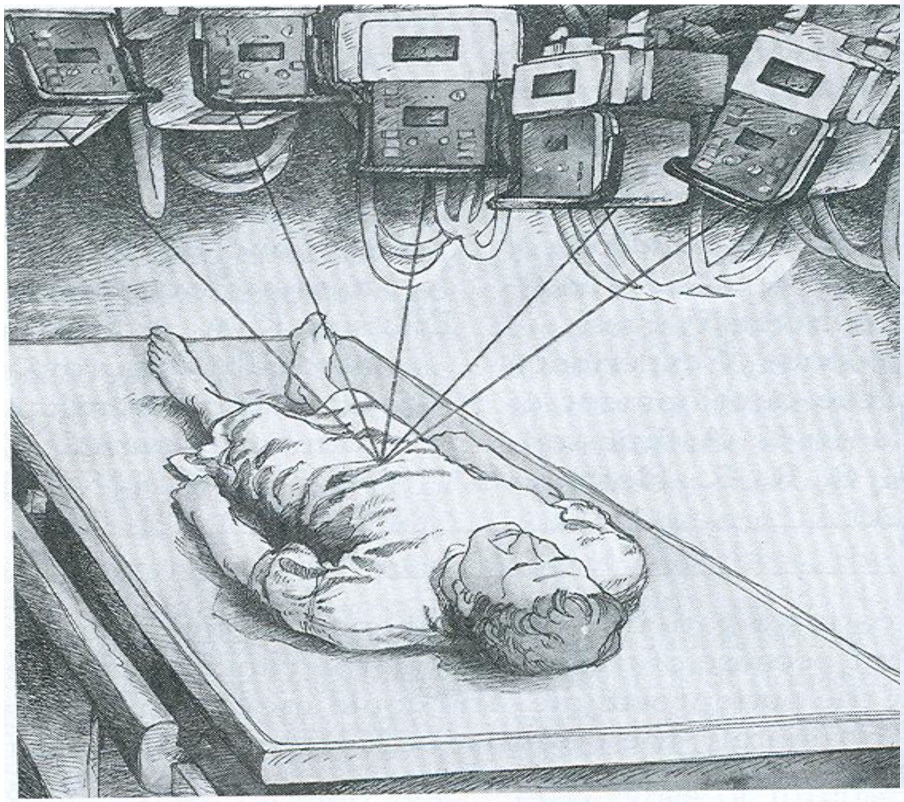
Gick and Holyoak Fortress Problem

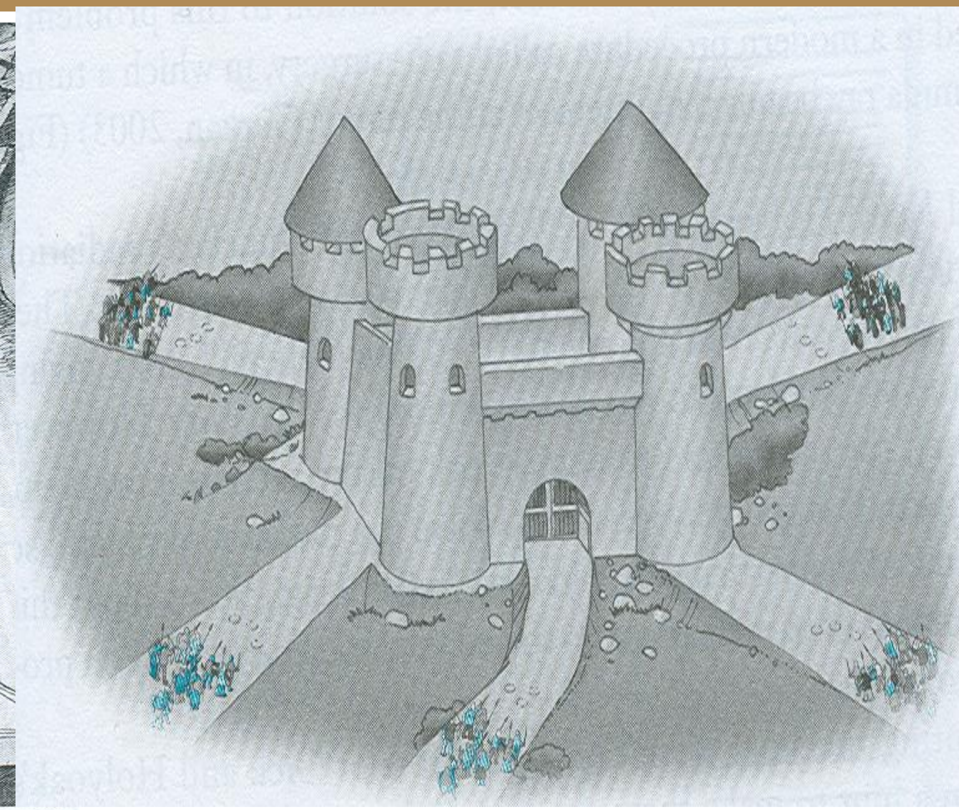
- **Passage 2 (Fortress Problem)**

- There was a small country ruled by a dictator staying in a fortress in the middle of the city. Many roads led to the fortress through the countryside. A General vowed to capture the fortress and overthrow the dictator. It was certain that his entire army could overpower the fortress if they should attack. However, the General learnt that the dictator had planted mines on each of the roads leading to the fortress. The mines were set such that small bodies of men could pass over safely without detonation. However, any large force will detonate the mines. Not only will the mines blow up the roads but may also destroy many neighboring villages. However, the general devised a simple plan. He divided his army into small groups and dispatched each group to the city center using a different road. When he was ready he gave the order for the groups to meet at the same time at the city center. In this way the general captured the fortress and overthrew the dictator.

- **Created by Gick and Holyoak (1980).**

- I am sure you have understood the two passages.
- In what way are the two passages related?
- If you read it carefully, you would realise that **Passage 2** *(Fortress Problem)* is the solution to **Passage 1***(Tumour Problem).*
- *See the demonstration of the solution below:*





2. **Fortress Problem**

1. **Tumour Problem**

Hill-Climbing Heuristic

Hill-Climbing Heuristic

- In this strategy, you make moves that reduce the distance between the current state and the goal state.

- Imagine your goal is to climb to the top of a hill. Just ahead of you, you meet a fork in the path and you cannot see ahead of you the distance on either path.

- To climb upward, you decide to use a path that has the steepest incline. In other words, you select the alternative that seems to lead to the solution or the goal state. Like many heuristics, the hill-climbing can lead you astray.

- The biggest drawback so far is that problem solvers must consistently make choices that appear to lead to the goal. In doing so there could be failures which lead to frustration.

Sample Questions

1. In problem solving, how do algorithms differ from heuristics? When you solve problems, what situations encourage which of these two approaches?
2. List and describe briefly the various problem solving approaches. Look for their similarities and differences.
3. Identify a time when you used the hill-climbing heuristic, and explain whether it was effective in solving the problem.

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