POLI 443
Applied Political Research

Session 4 – Tests of Hypotheses –The Normal Curve

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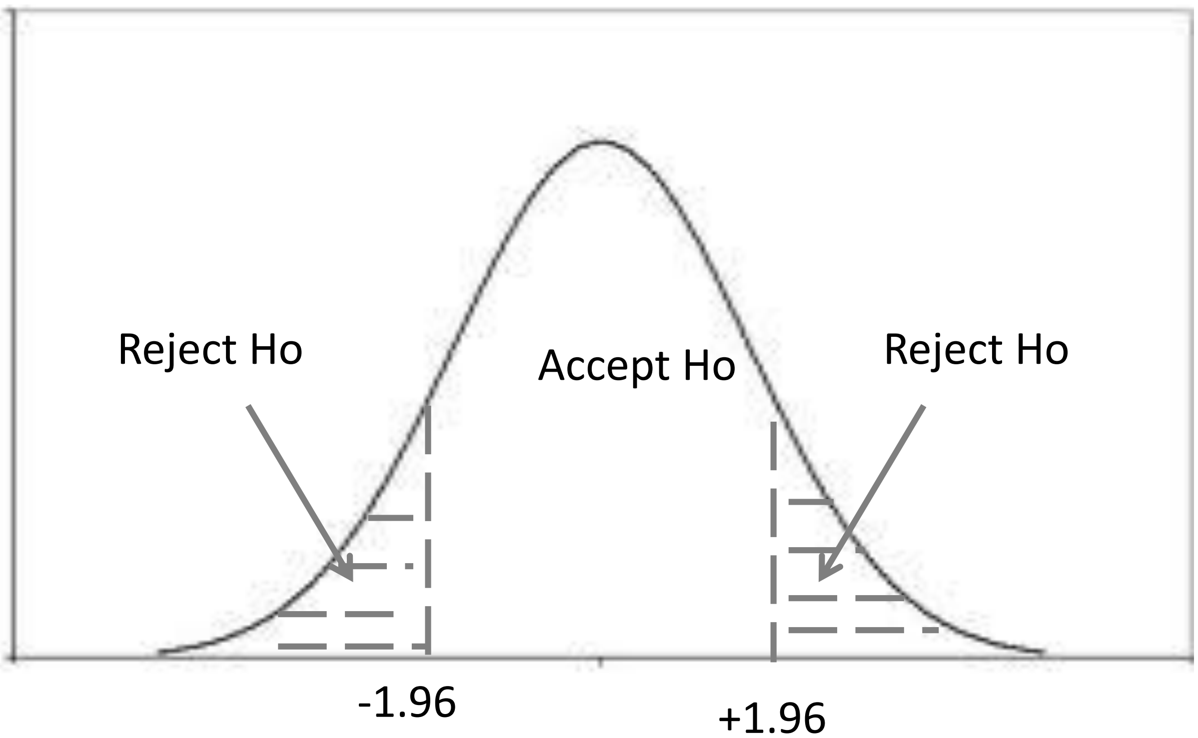
Test for the Normal Distribution

- The test of hypothesis is the scientific way of making decisions and the first of these tests will begin with the test for the normal distribution.
- Supposing it is claimed that the average age of University of Ghana students is 20 years, but others think it is higher. A sample of students' age is taken and the sample mean computed is 25 years. How different is 20 from 25? Assuming that the students' ages are normally distributed, we can hypothesize that the average age of the students is 20 and that

- any difference is due to sampling error. Using a confidence level of 5% we can say with 95% confidence that the hypothesis is true i.e. our Z value will lie between +/- 1.96.

Illustration

- If on choosing our sample of students we find that the Z value lies outside the range +/- 1.96 we would conclude that such an event could happen with probability of only 0.05 if the hypothesis were true. We would then say that the Z score differed significantly from what is expected under the hypothesis and we would be inclined to reject the hypothesis.
- The total shaded area in the illustration below = 0.05 is the level of significance of the test.


Reject Ho
Accept Ho
Reject Ho
-1.96
+1.96

- It represents the probability of our being wrong in rejecting the hypothesis i.e. the probability of making a type I error. Therefore we say that the hypothesis is rejected at a 0.05 level of significance or that the Z score of the given sample statistic is significant at the 0.05 level of significance.
- The Z scores outside the range +/- 1.96 constitute the critical region or rejection region of the hypothesis or the region of significance. The Z scores inside the +/- 1.96 is the region of acceptance of the hypothesis or the region of non significance.

One Tailed and Two-Tailed Tests

- In the example noted above we were interested in all the values of the test statistic (Z score) on both sides of the mean i.e. on both tails of the distribution. For this reason such tests are called two tailed tests. Very often there is interest in the Z score values on only one side of the mean i.e. one tail of the distribution e.g. when testing the hypothesis that one process or class is better than another process or class. Such tests are called one tailed tests. In this case the critical region (CR) is to one side of the distribution.

Critical Regions (CR) for Significance Tests

Level o Significance	0.10	0.05	0.01	0.005	0.002
CR for Z for 1 tailed test	-1.28 or +1.28	-1.645 or +1.645	-2.33 or +2.33	-2.58 or +2.58	-2.88 or +2.88
CR for Z for 2 tailed test	-1.645 & +1.645	-1.96 & +1.96	-2.58 & +2.58	-2.81 & +2.81	-3.08 & +3.08

Illustration 1

- Over a period of 10 years the mean vocabulary score of freshmen at the University of Ghana was 72 with a standard deviation of 7. The mean vocabulary score of this year's freshmen is 75 for a group of 100 students. Using the 5% level of significance, determine whether this group is better on the vocabulary test than the preceding class.

Procedure

1. Set the Null (Ho) and Alternative Hypothesis (Ha)
2. Select the appropriate test statistic for the computation
3. Select the Critical region (CR) of acceptance of Ho
4. Apply the test statistic (formulae) using the sample data
5. On the basis of 1 & 4 draw conclusions

Solution

- Elements given: $\mu = 72$ $\bar{x} = 75$ $s = 7$ $N = 100$
- H_O: The sample of this year's freshmen come from the same population as the previous year i.e. $\bar{x} = \mu$
- H_1: The sample of this year's freshmen does not come from the same population as the previous freshmen population and is better than the previous freshmen group i.e. $\bar{x} > \mu$

Test Statistic = Z = $\overline{x} - \mu / \sigma(\overline{x})$

Where the SE of $\overline{x} = \sigma(\overline{x}) = \sigma\sqrt{N}$

$= 7/\sqrt{10} = 0.7$

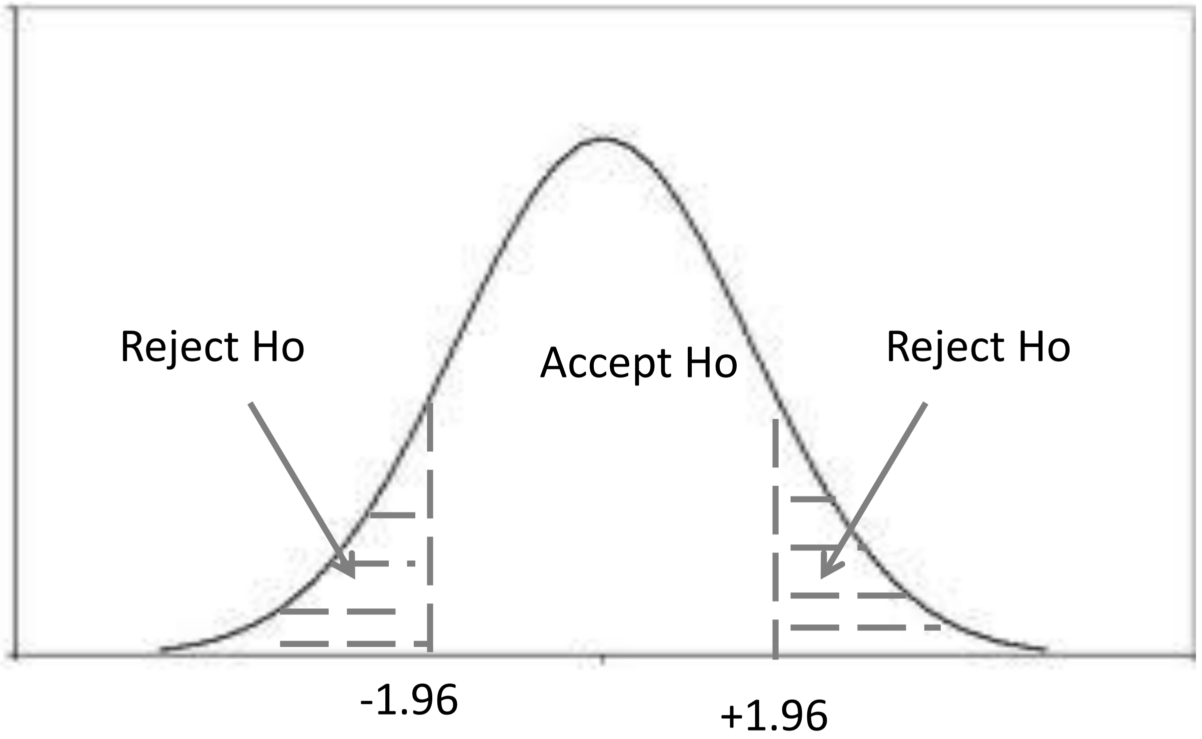
$Z = \overline{x} - \mu / \sigma(\overline{x})$

$= 75 – 72/0.7$

$= 3/0.7$

$= 30/7$

4.29


Reject Ho
Accept Ho
Reject Ho
-1.96
+1.96

- **Decision Rule:** At the 5% level Accept H_O if Z_C (Z calculated) lies within +/- 1.96
- **Conclusion:** Since the Z score = 4.29 is greater than (>) 1.96 it lies outside the region of acceptance of H_O. This means the result is significant at the 5% level of probability. We therefore reject the H_O and state that this year's freshmen class did significantly better in vocabulary than the classes preceding it over the past 10 years.

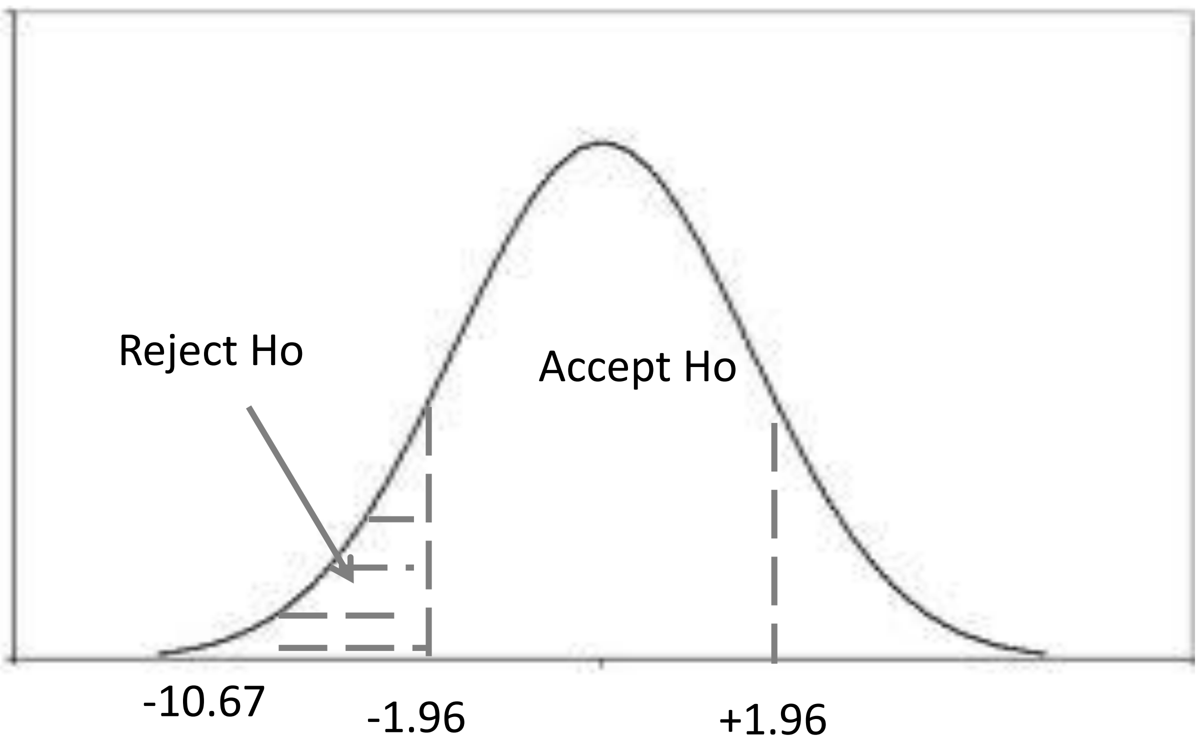
Illustration 2

- At the University of Lagos, the mean age of 50 Political Science major students was found to be 19.2 years while the mean age of all students at the University is 22.4 years. If the standard deviation of the age distribution of the Political Science major students is 2.1, is this an indication that the Political Science major students are younger than the rest of the students of the University? Test at the 0.05 level.

Solution

- Elements given: $\mu = 22.4$, $\bar{x} = 19.2$, $S = 2.1$ $N = 50$
- H_O: The Political Science major students have the same mean age as the rest of the students at the University i.e. $\bar{x} = \mu$
- H_1: The mean age of the Political Science major students is not the same as the rest of the students at the University i.e. $\bar{x} \neq \mu$
- Test Statistic = $Z = \bar{x} - \mu/\sigma(\bar{x})$
- Since is unknown we estimate it by use of S therefore we lose 1 df

- Therefore $\sigma(\overline{x}) = s/\ \sqrt{N} -1$
- $= 2.1/ \sqrt{50} -1$
- $=2.1/ \sqrt{49}$
- $= 2.1/7$
- $= 0.3$
- $Z = \overline{x}\ - \mu/\sigma(\overline{x})$
- $= 19.2 – 22.4/0.3$
- $= -3.2/0.3$
- $= - 10.67$
- **Decision Rule:** At the 5% level Accept H_O if Z_C (Z calculated) is within +/- 1.96


Reject Ho
Accept Ho
-10.67
-1.96
+1.96

- **Conclusion:**

  Since the Z score = -10.67 is greater than (>) +/-1.96 it lies outside the region of acceptance of H_O. This means the result is significant at the 5% level of probability. We therefore reject the H_O and conclude that the Political Science major students are younger than the rest of the students of the University.

Summary

- In this section you have learned to do tests involving the normal distribution in scientific decision making.

THANK YOU